Property (T) in simplicial complexes and the spectral evolution of random graphs

Elliot Paquette, The Ohio State University

This talk will introduce property (T) in the context of random simplicial complexes. This is a strong notion of expansion, expressed in terms of the fundamental group of the complex. Above the threshold for homological triviality, the fundamental group of the 2-dimensional Linial-Meshulam complex has (T). Below the threshold for homological triviality, it fails to have (T) but for trivial reasons: specifically, the group decomposes as a free-product of a giant (T) group and cyclic groups. This suggests there could be a much lower transition, at which there appears a giant-(T)-component. We show how to use the local spectral method to prove the existence of a small window below the homological triviality threshold in which this giant-(T) component exists.

This relies on new and optimal estimates on the spectrum of the normalized Laplacian of Erdos-Renyi graphs near the connectivity threshold.